THOMAS' CALCULUS (12/E)

7.1 Inverse Function and Their Derivatives

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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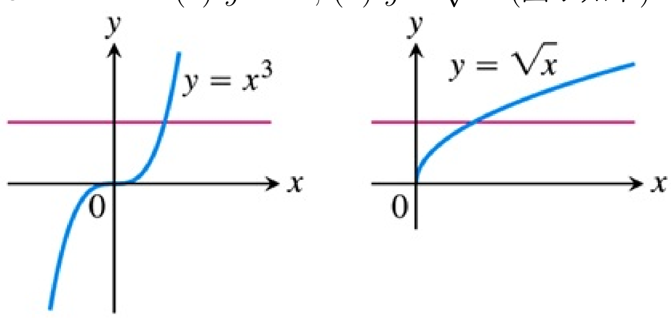
1 One-to-One Functions and Inverse Functions

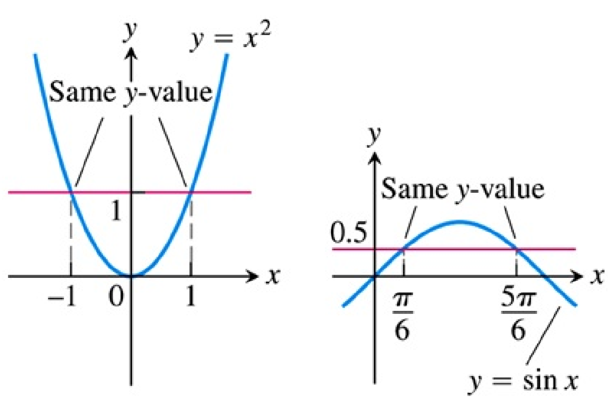
1.1 *Definitions: One-to-One Function*

> A function $f(x)$ is one-to-one on a domain D if ________ whenever ________ in D.

1.2 One-to-one: (a) $y = x^3$, (b) $y = \sqrt{x}$. (圖示如下)



1.3 Not one-to-one: (c) $y = x^2$, (d) $y = \sin x$. (圖示如下)



1.4 A function $y = f(x)$ is one-to-one if and only if its graph interests each ________________ at most ________.

1.5 *Definitions: Inverse Function*

> Suppose that f is a one-to-one function on a domain D with range R. The inverse function _______ is defined by
>
> _____________ if ___________
>
> The __________ of f^{-1} is ______ and the _________ of f^{-1} is ______.

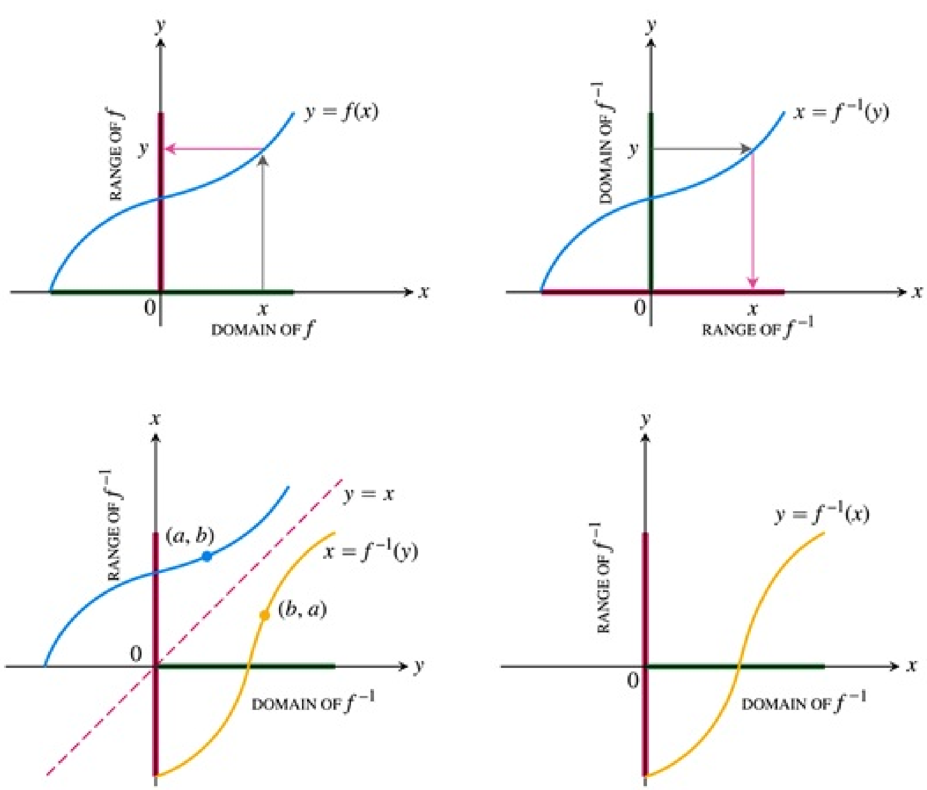
1.6 $(f^{-1} \cdot f)(x) =$ ______ for all x in the domain of f.

1.7 $(f \cdot f^{-1})(y) =$ ______ for all y in the domain of f^{-1}.

1.8 Only a one-to-one function can have an __________.

2 Finding Inverses

2.1 Determining the graph of $y = f^{-1}(x)$ from the graph of $y = f(x)$. (圖示如下)



2.2 Pass from f to f^{-1}.

(a) Solve the equation ____________ for x. This gives a formula ______________ where x is expressed as a function of y.

(b) Interchange ___________, obtaining a formula ______________ where f^{-1} is expressed in the conventional format with x as the _______________ variable and y as the _____________________.

Ex. **1** .... (example3, p364)

Find the inverse of $y = \frac{1}{2}x + 1$, expressed as a function of x.

sol:

Ex. **2** .... (example4, p364)

Find the inverse of the function $y = x^2,\ x \geq 0$, expressed as a function of x.

sol:

3 Derivatives of Inverses of Differentiable Functions

3.1 $f(x) = (1/2)x + 1$ and $f^{-1}(x) =$__________.

$$\frac{d}{dx}f(x) = \underline{\hspace{4cm}}$$

$$\frac{d}{dx}f^{-1}(x) = \underline{\hspace{4cm}}$$

3.2 *Theorem 1: The Derivative Rule for Inverses*

(a) If f has an interval I as domain and $f'(x)$ exists and is never zero on I, then f^{-1} is ________________ at every point in its domain.

(b) The value of $(f^{-1})'$ at a point b in the domain of f^{-1} is the _____________ of f' the value of at the point $a = f^{-1}(b)$:

$$(f^{-1})'(b) = \_\_\_\_\_\_\_\_\_\_ \quad \text{or} \quad \frac{d}{dx}f^{-1}\bigg|_{x=b} = \_\_\_\_\_\_\_\_\_\_$$

3.3 When $y = f(x)$ is differentiable at $x = a$ and we change x by a small amount dx, the corresponding change in y is approximately ________________. This means that y changes about _________ times as fast as x when $x = a$ and that x changes about ___________ times as fast as y when $y = b$.

3.4 It is reasonable that the derivative of f^{-1} at b is the _____________ of the derivative of f at a.

Ex. 3 .... (`example5, p366`)

Apply The Derivative Rule for Inverse Theorem to the function $f(x) = x^2$, $x \geq 0$.

sol:

Ex. 4 .... (`example6, p366`)

Let $f(x) = x^3 - 2$. Find the value of df^{-1}/dx at $x = 6 = f(2)$ without finding a formula for $f^{-1}(x)$.

sol:

實習課練習 (EXERCISE 7.1)

21. Let $f(x) = x^3 - 1$. Find a formula for f^{-1}.

22. Let $f(x) = x^2 - 2x + 1,\ x \geq 1$. Find a formula for f^{-1}.

33. Let $f(x) = x^2 - 2x,\ x \leq 1$. Find f^{-1} and identify the domain and range of f^{-1}.

37. Let $f(x) = 5 - 4x,\ a = 1/2$. Find $f^{-1}(x)$. Evaluate df/dx at $x = a$ and df^{-1}/dx at $x = f(a)$.

38. Let $f(x) = 2x^2,\ x \geq 0,\ a = 5$. Find $f^{-1}(x)$. Evaluate df/dx at $x = a$ and df^{-1}/dx at $x = f(a)$.

41. Let $f(x) = x^3 - 3x^2 - 1,\ x \geq 2$. Find the value of df^{-1}/dx at the point $x = -1 = f(3)$.