THOMAS' CALCULUS (12/E)

7.2 Natural Logarithms

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Definition of the Natural Logarithm Function

1.1 The natural logarithm of a positive number x, written as ________, is the value of an integral.

1.2 *Definitions: The Natural Logarithm Function*

$$\ln x = \underline{\qquad\qquad\qquad}, \quad x > 0$$

1.3 If $x > 1$, then $\ln x$ is the area under the curve __________ from _________ to _________.

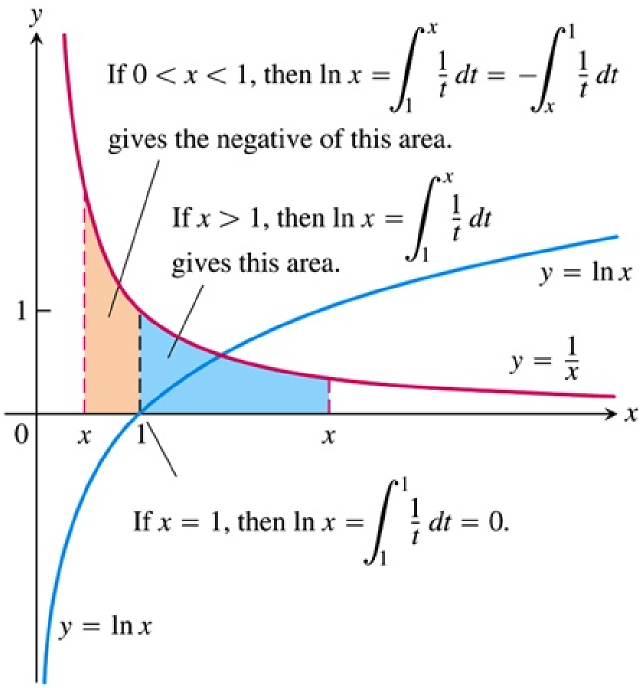
1.4 For $0 < x < 1$, $\ln x$ gives the ____________________ under the curve from ______ to ______.

1.5 *Definitions: The Number e*

The number e is that number in the domain of the natural logarithm satisfying ____________.

1.6 $\ln 1 = \underline{\qquad\qquad\qquad}$.

1.7 The graph of $y = \ln x$ and its relation to the function $y = 1/x,\ x > 0$.
The graph of the logarithm rises above the x-axis as x moves from 1 to the right, and it falls below the axis as x moves from 1 to the left. (圖示如下)



1.8 Geometrically, the number e corresponds to the point on the x-axis for which the area under the graph of __________ and above the interval ________ is the exact area of the unit square.

2 The Derivative of $y = \ln x$

2.1 $\frac{d}{dx} \ln x =$ ______________ $=$ ______

2.2 $y = \ln u$

$$\frac{d}{dx} \ln u = \_\_\_\_\_\_ = \_\_\_\_\_\_, \quad u > 0$$

Ex. 1 .... (example1, p371)

1. $\frac{d}{dx} \ln 2x =$
2. $\frac{d}{dx} \ln(x^2 + 3) =$
3. $\frac{d}{dx} \ln ax =$

3 Properties of Logarithms

3.1 *Theorem 2: Properties of Logarithms*

For any numbers $a > 0$ and $x > 0$, the natural logarithm satisfies the following rules:

(a) Product Rule: ______________

(b) Quotient Rule: ______________

(c) Reciprocal Rule: ______________

(d) Power Rule: ______________

Ex. 2 .... (example2, p372)

1. $\ln 4 + \ln \sin x =$
2. $\ln \dfrac{x+1}{2x-3} =$
3. $\ln \sec x =$
4. $\ln \sqrt[3]{x+1} =$

4 The Integral $\int (1/u) du$

4.1 If u is a differentiable function that is never zero,

$$\int \frac{1}{u}\, du = \_\_\_\_\_\_\_\_\_\_.$$

4.2 If $u = f(x)$, then $du =$__________ and

$$\int \frac{1}{u}\, du = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_.$$

Ex. 3 .... (example3, p374)

$$\int_0^2 \frac{2x}{x^2 - 5}\, dx$$

sol:

5 The Integral of $\tan x$, $\cot x$, $\sec x$ and $\csc x$

5.1 $\int \tan u \; du =$ ______________

Proof:

5.2 $\int \cot u \; du =$ ______________

Proof:

5.3 $\int \sec u \; du =$ ____________________

Proof:

5.4 $\int \csc u \; du =$ ______________________

Proof:

Ex. **4** .... (example4, p375)

$$\int_0^{\pi/6} \tan 2x \ dx$$

sol:

6 Logarithmic Differentiation

✐ Ex. **5** .... (`example5, p375`)

Find dy/dx if $y = \dfrac{(x^2+1)(x+3)^{1/2}}{x-1}, \quad x > 1.$

sol:

實習課練習 (EXERCISE 7.2)

2. Express the following logarithms in terms of $\ln 5$ and $\ln 7$. (a) $\ln(1/125)$, (b) $\ln 9.8$, (c) $\ln 7\sqrt{7}$, (d) $\ln 1225$, (e) $\ln 0.056$, (f) $(\ln 35 + \ln(1/7))/(\ln 25)$.

3. Simplify the expressions: (a) $\ln \sin\theta - \ln(\frac{\sin\theta}{5})$, (b) $\ln(3x^2 - 9x) + \ln(\frac{1}{3x})$, (c) $\frac{1}{2}\ln(4t^4) - \ln 2$.

□ Find the derivative of y with respect to x, or t, as appropriate.

8. $y = \ln(t^3/2)$

15. $y = t(\ln t)^2$

22. $y = \dfrac{x \ln x}{1 + \ln x}$

35. $y = \displaystyle\int_{x^2/2}^{x^2} \ln\sqrt{t}\ dt$

□ Evaluate the integrals:

38. $\displaystyle\int_{-1}^{0} \frac{3}{3x - 2}\ dx$

44. $\displaystyle\int_{2}^{4} \frac{dx}{x \ln x}$

45. $\displaystyle\int_{2}^{4} \frac{dx}{x(\ln x)^2}$

52. $\displaystyle\int_{0}^{\pi/12} 6\tan 3x\ dx$

53. $\displaystyle\int \frac{dx}{2\sqrt{x} + 2x}$

□ Use logarithmic differentiaton to find the derivative of y with respect to the given independent variable.

55. $y = \sqrt{x(x+1)}$

62. $y = \dfrac{1}{t(t+1)(t+2)}$

67. $y = \sqrt[3]{\dfrac{x(x-2)}{x^2+1}}$