

THOMAS' CALCULUS (12/E)

7.6 Inverse Trigonometric Functions

開課班級: (105-2) 通訊1/電機1/智財學程 微積分

授課教師: 吳漢銘 (國立臺北大學統計學系 副教授)

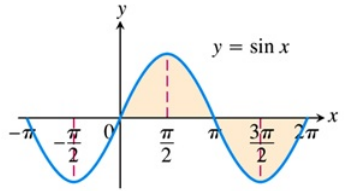
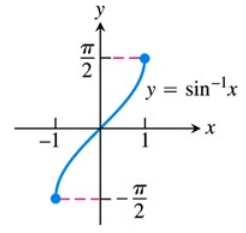
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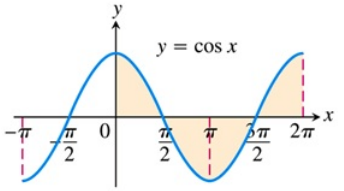
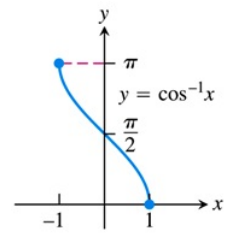
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1 Defining the Inverses

1.1 Domain restrictions that make the trigonometric functions one-to-one.

(a) $y = \sin^{-1} x$ is the number in _____ for which _____.(b) $y = \cos^{-1} x$ is the number in _____ for which _____.

$\sin x$	$\sin^{-1} x$ or $\arcsin x$
D: _____ R: _____ 	D: _____ R: _____ 

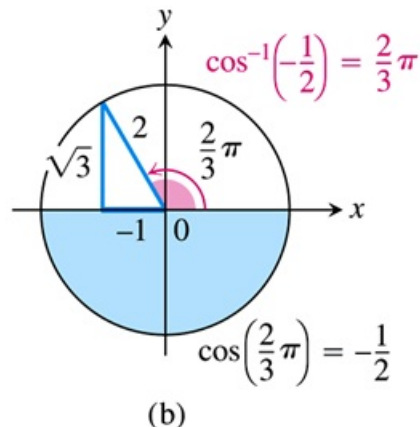
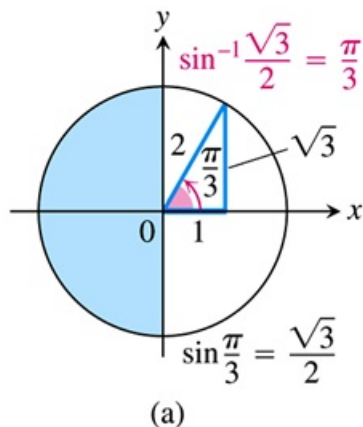
$\cos x$	$\cos^{-1} x$ or $\arccos x$
D: _____ R: _____ 	D: _____ R: _____ 

$\tan x$	$\tan^{-1} x$ or $\arctan x$
D: _____ R: _____	D: _____ R: _____

$\cot x$	$\cot^{-1} x$ or $\operatorname{arccot} x$
D: _____ R: _____	D: _____ R: _____

$\sec x$	$\sec^{-1} x$ or $\operatorname{arcsec} x$
D: _____ R: _____	D: _____ R: _____

$\csc x$	$\csc^{-1} x$ or $\operatorname{arccsc} x$
D: _____ R: _____	D: _____ R: _____

1.2 Common values of $\sin^{-1} x$:

x	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	$-1/2$	$-\sqrt{2}/2$	$-\sqrt{3}/2$
$\sin^{-1} x$						
$\cos^{-1} x$						

TABLE 1.3 Values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ for selected values of θ

Degrees	-180	-135	-90	-45	0	30	45	60	90	120	135	150	180	270	360
θ (radians)	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin \theta$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\cos \theta$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan \theta$	0	1		-1	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0		0

2 The Derivative of Inverse Trigonometric Functions

2.1 $(f(x)^{-1})' =$ _____ .

2.2 Find the derivative of $y = \sin^{-1} x$.

2.3 Find the derivative of $y = \tan^{-1} x$.

2.4 Find the derivative of $y = \sec^{-1} x$.

2.5 $u = u(x)$

(a) $\frac{d}{dx}(\sin^{-1} u) = \underline{\hspace{2cm}}$, $|u| < 1$.

(b) $\frac{d}{dx}(\tan^{-1} u) = \underline{\hspace{2cm}}$.


(c) $\frac{d}{dx}(\sec^{-1} u) = \underline{\hspace{2cm}}$.

2.6 *Inverse Function-Inverse Cofunction Identities*

(a) $\cos^{-1} x = \underline{\hspace{2cm}} \Rightarrow \frac{d}{dx}(\cos^{-1} u) = \underline{\hspace{2cm}}$, $|u| < 1$


(b) $\cot^{-1} x = \underline{\hspace{2cm}} \Rightarrow \frac{d}{dx}(\cot^{-1} u) = \underline{\hspace{2cm}}$

(c) $\csc^{-1} x = \underline{\hspace{2cm}} \Rightarrow \frac{d}{dx}(\csc^{-1} u) = \underline{\hspace{2cm}}$, $|u| > 1$

 **Ex. 1** (example4, p408)

$$\frac{d}{dx}(\sin^{-1} x^2) =$$

sol:

 **Ex. 2** (example5, p410)

$$\frac{d}{dx}(\sec^{-1} 5x^4) =$$

sol:

3 Integration Formulas

$$3.1 \int \frac{du}{\sqrt{a^2 - u^2}} = \underline{\hspace{2cm}}, \quad u^2 < a^2$$

$$3.2 \int \frac{du}{a^2 + u^2} = \underline{\hspace{2cm}}$$


$$3.3 \int \frac{du}{u\sqrt{u^2 - a^2}} = \underline{\hspace{2cm}}, \quad |u| > a > 0$$

 **Ex. 3** (example6, p412)

$$(a) \int_{\sqrt{2}/2}^{\sqrt{3}/2} \frac{dx}{\sqrt{1-x^2}} =$$


$$(b) \int \frac{dx}{\sqrt{3-4x^2}} =$$

$$(c) \int \frac{dx}{\sqrt{e^{2x} - 6}} =$$

 Ex. 4 (example7(a), p412)

Evaluate $\int \frac{dx}{\sqrt{4x - x^2}}$

sol:

 Ex. 5 (example7(b), p412)

Evaluate $\int \frac{dx}{4x^2 + 4x + 2}$

sol:

實習課練習 (EXERCISE 7.6)

□ Find the values.

9. $\sin(\cos^{-1}(\frac{\sqrt{2}}{2}))$

12. $\cot(\sin^{-1}(-\frac{\sqrt{3}}{2}))$

□ Find the derivative of y with respect to the appropriate variable.

22. $y = \cos^{-1}(1/x)$ $y = \sin^{-1}(3/t^2)$

33. $y = \ln(\tan^{-1} x)$

41. $y = x \sin^{-1} x + \sqrt{1-x^2}$

42. $y = \ln(x^2 + 4) - x \tan^{-1}(\frac{x}{2})$

□ Evaluate the integrals.

44. $\int \frac{dx}{\sqrt{1-4x^2}}$

46. $\int \frac{dx}{9+3x^2}$

47. $\int \frac{dx}{x\sqrt{25x^2-2}}$

63. $\int_0^{\ln \sqrt{3}} \frac{e^x dx}{1+e^{2x}}$

68. $\int \frac{dx}{\sqrt{2x-x^2}}$

72. $\int \frac{dy}{y^2+6y+10}$

79. $\int \frac{dx}{(x+1)\sqrt{x^2+2x}}$

83. $\int \frac{(\sin^{-1} x)^2 dx}{\sqrt{1-x^2}}$

86. $\int \frac{dy}{(\sin^{-1} y)\sqrt{1-y^2}}$