THOMAS' CALCULUS (12/E)

8.4 Integration of Rational Functions by Partial Fractions

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1 Partial Fractions

1.1 This section shows how to express a rational function (a quotient of polynomials) as a sum of simpler fractions, called _____,

Example: $\dfrac{5x-3}{x^2-2x-3} =$ _____

$\displaystyle\int \frac{5x-3}{x^2-2x-3} =$ _____ = _____

1.2 The method of partial fractions: Example: $\dfrac{5x-3}{x^2-2x-3} =$ _____

✎ Ex. 1 ..... (example1, p455)

Use partial fractions to evaluate $\displaystyle\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)}\,dx$ (Distinct linear factors)

sol:

✎ Ex. **2** ..... (example2, p455)

Use partial fractions to evaluate $\int \frac{6x+7}{(x+2)^2}\,dx$ (A repeated linear factor)

sol:

✎ Ex. **3** ..... (example3, p456)

Use partial fractions to evaluate $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx$ (An improper fraction)

sol:

✎ Ex. **4** ..... (example4, p456)

Use partial fractions to evaluate $\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$ (An irreducible quadratic factor)

sol:

✎ Ex. 5 ..... (example5, p457)

Use partial fractions to evaluate $\int \frac{dx}{x(x^2+1)^2}\,dx$ (An repeated irreducible quadratic factor)

sol:

2 The Heaviside Method for Linear Factors

2.1 *Heaviside Method*

Write the partial-fraction expansion of $f(x)/g(x)$ as

$$\frac{f(x)}{g(x)} = \underline{\qquad\qquad\qquad\qquad},$$

where

$$A_1 = \underline{\qquad\qquad}$$

$$A_2 = \underline{\qquad\qquad\qquad}$$

$$\vdots$$

$$A_n = \underline{\qquad\qquad\qquad\qquad}$$

✎ Ex. 6 ..... (example6, p458)

Find A, B and C in the equation: $\frac{x^2+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

sol:

Ex. 7 ..... (example7, p459)

Use the Heaviside Method to evaluate $\int \frac{x+4}{x^3+3x^2-10x}\,dx$.

sol:

Ex. 8 ..... (example8, p460)

Find A, B and C in the equation: $\frac{x-1}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$

sol:

✎ Ex. **9** ..... (example9, p460)

Find A, B and C in the expression: $\dfrac{x^2+1}{(x-1)(x-2)(x-3)} = \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$

sol:

實習課練習 (EXERCISE 8.4)

☐ Expand the quotients by partial fractions.

2. $\dfrac{5x-7}{x^2-3x+2}$

5. $\dfrac{z+1}{z^2(z-1)}$

8. $\dfrac{t^4+9}{t^4+9t^2}$

. . . . . .

☐ Express the integrands as a sum of partial fractions and evaluate the integrals.

16. $\displaystyle\int \frac{x+3}{2x^3-8x}\,dx$

17. $\displaystyle\int_0^1 \frac{x^3\,dx}{x^2+2x+1}$

20. $\displaystyle\int \frac{x^2\,dx}{(x-1)(x^2+2x+1)}$

24. $\displaystyle\int \frac{8x^2+8x+2}{(4x^2+1)^2}\,dx$

33. $\displaystyle\int \frac{2x^3-2x^2+1}{x^2-x}\,dx$

39. $\displaystyle\int \frac{e^t\,dt}{e^{2t}+3e^t+2}$

42. $\displaystyle\int \frac{\sin\theta\,d\theta}{\cos^2\theta+\cos\theta-2}$