THOMAS' CALCULUS (12/E)

# 10.2 Infinite Series

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## 1 Series

1.1 *Definitions: Infinite Series, nthTerm, Partial Sum, Converges, Sum*

(a) Given a sequence of numbers $\{a_n\}$, an expression of the form

____________________

is an __________. The number $a_n$ is the ________ of the series.

(b) The sequence $\{s_n\}$ defined by

$$s_n = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_$$

is the sequence of __________ of the series, the number being the _____ partial sum.

(c) If ________________ converges to a limit $L$, we say that ____________ and that its sum is ____. In this case, we also write

$$a_1 + a_2 + \cdots + a_n + \cdots = \_\_\_\_\_\_\_\_\_\_\_\_$$

(d) If the sequence of partial sums of the series does not converge, we say that the series _______.

1.2 The series of the form

$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$

is called __________, in which $a$ and $r$ are fixed real number and $a \neq 0$.

(a) Examples: $a = 1$

i. $r = \frac{1}{2}$,

ii. $r = -\frac{1}{3}$,

iii. $r = 1$

(b) $s_n =$

(c) If $|r| < 1$, the geometric series converges: ____________________, $|r| < 1$.

(d) If $|r| \geq 1$, the series ____________.

✎ Ex. **1** ............................................... (example1, p546)

$$\sum_{n=1}^{\infty} \frac{1}{9}(\frac{1}{3})^{n-1} =$$

✎ Ex. **2** ............................................... (example2, p546)

$$\sum_{n=0}^{\infty} \frac{(-1)^n 5}{4^n} =$$

✎ Ex. **3** ............................................... (example4, p547)

Express the repeating decimal 5.232323... as the ratio of two integers.

*sol:*

**Ex. 4** ............................................ (example5, p547)

Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

*sol:*

# 2 The $n$th-Term Test for Divergence

2.1 *Theorem 7*

If ____________ converges, then ___________.

2.2 *The nth-Term Test for Divergence*

$\displaystyle\sum_{n=1}^{\infty} a_n$ diverges if ______________ fails to _________ or is different from _________.

2.3 *Theorem 8*

If $\sum a_n = A$ and $\sum b_n = B$ are convergent series, then

(a) Sum Rule: ______________________________________

(b) Difference Rule: ______________________________________

(c) Constant Multiple Rule: _________________________, (any number $k$)

2.4 $\sum(a_n + b_n)$ can converge when $\sum a_n$ and $\sum b_n$ diverge.
Example: $\sum a_n =$ ______________________, $\sum b_n =$ ____________________________,
$\sum(a_n + b_n) =$ __________________

2.5 Adding or Deleting Terms: we can add or delete a __________ number of terms without altering the series'convergence or divergence, although in the case of convergence this will usually change the sum.

2.6 Reindexing:

(a) $\sum_{n=1}^{\infty} a_n =$ ______________________________

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then ____________ converges for any $k > 1$.

(c) If $\sum_{n=k}^{\infty} a_n$ converges for any $k > 1$ then ____________ converges.

(d) $\sum_{n=1}^{\infty} a_n =$ _______________ $=$ _______________ $=$ ___________________

(e) Example:

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \underline{\hspace{3cm}} = \underline{\hspace{3cm}} = \underline{\hspace{3cm}}$$

**Ex. 5** ............................................ (example7, p548)

Test the divergences of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} n^2$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{-n}{2n+5}$

**Ex. 6** ............................................ (example9, p550)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}} =$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{2^n} =$

# 實習課練習 (EXERCISE 10.2)

☐ Find the sum of the series if the series converges.

**7.** $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$.

**13.** $\sum_{n=0}^{\infty} (\frac{1}{2^n} + \frac{(-1)^n}{5^n})$.

**44.** $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$.

**45.** $\sum_{n=1}^{\infty} \left( \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$

**47.** $\sum_{n=1}^{\infty} \left( \frac{1}{\ln(n+2)}) - \frac{1}{\ln(n+1)} \right)$

☐ Express each of the number as the ratio of two integers.

**20.** $0.\overline{234} = 0.234234234...$

**25.** $1.24\overline{123} = 1.24123123...$

☐ Use the $n$th-Term Test for divergence to show that the series is divergent, or state that the test is inconclusive.

**30.** $\sum_{n=1}^{\infty} \frac{n}{n^2+3}$

**33.** $\sum_{n=1}^{\infty} \ln \frac{1}{n}$

☐ Which series converge and which diverge? If a series converges, find its sum.

**49.** $\sum_{n=0}^{\infty} (\frac{1}{\sqrt{2}})^n$

**54.** $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$

**58.** $\sum_{n=0}^{\infty} \frac{1}{x^n}, \quad |x| > 1.$

**60.** $\sum_{n=1}^{\infty}(1-\frac{1}{n})^n$

**67.** $\sum_{n=0}^{\infty}(\frac{e}{\pi})^n$

□ Find the values of $x$ for which the given geometric series converges.

**75.** $\sum_{n=0}^{\infty}(-1)^n(x+1)^n$

**78.** $\sum_{n=0}^{\infty}(\ln x)^n$