

THOMAS' CALCULUS (12/E)

10.3 The Integral Test

開課班級: (105-2) 通訊1/電機1/智財學程 微積分

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1 Nondecreasing Partial Sums

1.1 Given a series we have two questions: (a) Does the series _____? (b) If it converges, what is its _____?

1.2 Suppose that $\sum_{n=1}^{\infty} a_n$ is an infinite series with $a_n \geq 0$ for all n . Then each partial sum is greater than or equal to its predecessor because _____. The partial sums form a _____:

1.3 *Corollary of Theorem 6*

A series $\sum_{n=1}^{\infty} a_n$ of nonnegative terms converges if and only if its _____ are _____.

1.4 The _____

$$\sum_{n=1}^{\infty} \frac{1}{n} = \underline{\hspace{10em}}$$


is divergent, but this doesn't follow from the n th-Term Test.

The n th term $1/n$ does go to zero, but the series still diverges. The reason it diverges is because there is no upper bound for its partial sums.

1.5 *Theorem 9: The Integral Test*


Let $\{a_n\}$ be a sequence of _____. Suppose that _____, where f is a _____, _____, _____ function of x for all $x \geq N$ (N a positive integer). Then the series _____ and the integral _____ both converge or both diverge.

(說明如下:)

 **Ex. 1** (example2, p553)

Does $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges?

sol:


 **Ex. 2** (example3, p555)

Show that the **p-series**

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$$

converges if $p > 1$, and diverges if $p \leq 1$.

sol:

 **Ex. 3** (example4, p555)

Does $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$ converge?

sol:

實習課練習 (EXERCISE 10.3)

Use the Integral Test to determine if the series converge or diverge.

3. $\sum_{n=1}^{\infty} \frac{1}{n^2 + 4}$.

6. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

9. $\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$.

Determining convergence or divergence.

15. $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$.

19. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$.

22. $\sum_{n=1}^{\infty} \frac{5^n}{4^n + 3}$.

27. $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$.

28. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$.

30. $\sum_{n=1}^{\infty} \frac{1}{(\ln 3)^n}$.

36. $\sum_{n=1}^{\infty} \frac{2}{1 + e^n}$.

38. $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$.