

THOMAS' CALCULUS (12/E)

10.4 Comparison Tests

開課班級: (105-2) 通訊1/電機1/智財學程 微積分

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1 Comparison Tests

1.1 Theorem 10: The Comparison Test

Let $\sum a_n$ be a series with _____ terms.(a) $\sum a_n$ _____ if there is a _____ series $\sum c_n$ with _____ for all $n > N$, for some integer N .(b) $\sum a_n$ _____ if there is a _____ series of nonnegative terms $\sum d_n$ with _____ for all $n > N$, for some integer N . **Ex. 1** (example1, p559)(a) Does $\sum_{n=1}^{\infty} \frac{5}{5n-1}$ converge?(b) Does $\sum_{n=0}^{\infty} \frac{1}{n!}$ converge?

(c) Does $5 + \frac{2}{3} + \frac{1}{7} + 1 + \frac{1}{2 + \sqrt{1}} + \cdots + \frac{1}{2^n + \sqrt{n}} + \cdots$ converge?

2 The Limit Comparison Test

2.1 Theorem 11: Limit Comparison Test

Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N an integer).

(a) If _____ then _____ both converge or both diverge.

(b) If _____ and _____ converges, then _____ converges.


(c) If _____ and _____ diverges, then _____ diverges.

 **Ex. 2** (example2, p560)

(a) Does $\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$ converge?

(b) Does $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ converge?

(c) Does $\sum_{n=2}^{\infty} \frac{1 + n \ln n}{n^2 + 5}$ converge?

 **Ex. 3** (example3, p561)

Does $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converge?

sol:

實習課練習 (EXERCISE 10.4)

Use the Comparison Test to determine if each series converges or diverges.

1.
$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 30}.$$

6.
$$\sum_{n=1}^{\infty} \frac{1}{n3^n}.$$

Use the Limit Comparison Test to determine if each series converges or diverges.

10.
$$\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}.$$

14.
$$\sum_{n=1}^{\infty} \left(\frac{2n+3}{5n+4}\right)^n.$$

Determining convergence or divergence?

17.
$$\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + \sqrt[3]{n}}$$

19.
$$\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$$

28.
$$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^3}$$

34.
$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 1}$$

38.
$$\sum_{n=1}^{\infty} \frac{3^{n-1} + 1}{3^n}$$

51.
$$\sum_{n=1}^{\infty} \frac{1}{n \sqrt[n]{n}}$$

53.
$$\sum_{n=1}^{\infty} \frac{1}{1 + 2 + 3 + \cdots + n}$$