THOMAS' CALCULUS (12/E)

11.3 Polar Coordinates

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Definition of Polar Coordinates

1.1 *Definitions: Polar Coordinates*

Fix an _____ (called the _____) and an initial _____ from O. Then each point P can be located by assigning to it a _____ pair _____ in which r gives the _____ from O to P and θ gives the _____ from the initial ray to ray $\bar{OP}$.

1.2 圖示如下:

(a) Polar coordinates (b) (r, θ) are not unique. (c) (r, θ) can have negative r-values.

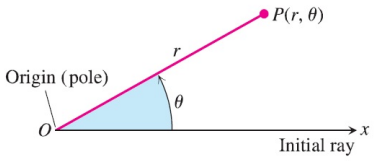

FIGURE 11.18 To define polar coordinates for the plane, we start with an origin, called the pole, and an initial ray.

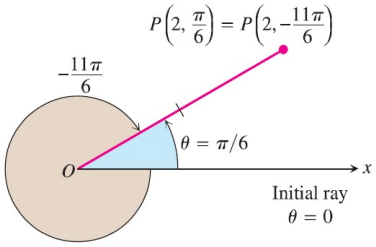

FIGURE 11.19 Polar coordinates are not unique.

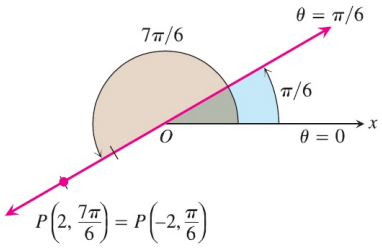

FIGURE 11.20 Polar coordinates can have negative r-values.

1.3 One pair of cartesian coordinates imples _____ pairs of polar coordinates.

1.4 Polar Equations and Graphs (I)

(a) If we hold r fixed at a constant value _____, the point _____ will lie _____ units from the origin O.

(b) As θ varies over any interval of length _____, P then traces a circle of _____ centered at O.

(c) If we hold θ fixed at a constant value _____ and let r vary between _____, the point $P(r, \theta)$ traces the line through O that makes an angle of measure θ_0 with the initial ray.

1.5 Polar Equations and Graphs (II)

(a) Equation: $r = a$, Graph: _____

(b) Equation: $\theta = \theta_0$, Graph: _____

(c) $r = 1$ and $r = -1$ are equations for the _____.

Ex. 1 ..... (example1, p627)

Find all the polar coordinates of the point $P(2, \pi/6)$

sol:

Ex. 2 ..... (example3, p628)

Graph the sets of points whose polar coordinates satisfy the following conditions.

(a) $1 \leq r \leq 2$ and $0 \leq \theta \leq \dfrac{\pi}{2}$

(b) $-3 \leq r \leq 2$ and $\theta = \dfrac{\pi}{4}$

(d) $\dfrac{2\pi}{3} \leq \theta \leq \dfrac{5\pi}{6}$

2 Relating Polar and Cartesian Coordinates

2.1 *Equations Relating Polar and Cartesian Coordinates*

$$x = r\cos\theta, \quad y = r\sin\theta, \quad x^2 + y^2 = r^2$$

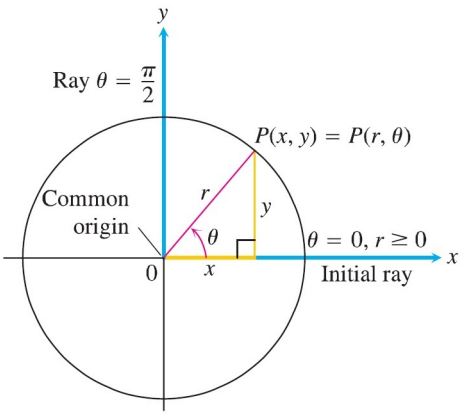

FIGURE 11.24 The usual way to relate polar and Cartesian coordinates.

Ex. 3 ..... (example5, p629)

Find a polar equation for the circle $x^2 + (y-3)^2 = 9$.

sol:

Ex. 4 ..... (example6, p629)

Replace the following polar equations by equivalent Cartesian equations, and identify their graphs.

(a) $r\cos\theta = -4$

(b) $r^2 = 4r\cos\theta$

(c) $r = \dfrac{4}{2\cos\theta - \sin\theta}$

sol:

實習課練習 (EXERCISE 11.3)

☐ Plot the following points given in a polar coordinates. Then find all the polar coordinates of each point.

3. $(2, \pi/2)$, $(2, 0)$, $(-2, \pi/2)$, $(-2, 0)$.

4. $(3, \pi/4)$, $(-3, \pi/4)$, $(3, -\pi/4)$, $(-3, -\pi/4)$.

☐ Replace the following polar equation by equivalent Cartesian equations.

30. $r\cos\theta = 0$

33. $r\cos\theta + r\sin\theta = 1$

38. $r^2 \sin 2\theta = 2$

44. $\cos^2\theta = \sin^2\theta$

49. $r = 2\cos\theta + 2\sin\theta$

☐ Replace the following Cartesian equation by equivalent polar equations.

54. $y = 1$

57. $x^2 + y^2 = 4$

60. $xy = 2$

62. $x^2 + xy + y^2 = 1$

65. $(x-3)^2 + (y+1)^2 = 4$