

THOMAS' CALCULUS (12/E)

11.5 Areas and Lengths in Polar Coordinates

開課班級: (105-2) 通訊1/電機1/智財學程 微積分

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1 Area in the Plane

1.1 The fan-shaped circular sector has radius _____ and central angle of radian measure _____. The area is _____ times the area of a circle of radius _____, or

$$A_k = \underline{\hspace{2cm}}, \quad \sum_{k=1}^n A_k = \underline{\hspace{2cm}}$$

$$A = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

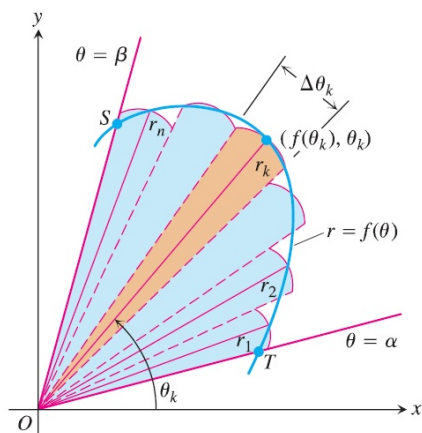


FIGURE 11.30 To derive a formula for the area of region OTS , we approximate the region with fan-shaped circular sectors.

1.2 Area of the Fan-Shaped Region Between the Origin and the Curve $r = f(\theta)$, $\alpha < \theta < \beta$

$$A = \int_{\alpha}^{\beta} \frac{1}{2} f(\theta)^2 d\theta .$$

This is the integral of the area differential $dA = \frac{1}{2} f(\theta)^2 d\theta = \frac{1}{2} r^2 d\theta$.


1.3 Area of the Region $0 \leq r_1(\theta) \leq r \leq r_2(\theta)$, $\alpha \leq \theta \leq \beta$

$$A = \int_{\alpha}^{\beta} \frac{1}{2} (r_2(\theta)^2 - r_1(\theta)^2) d\theta = \int_{\alpha}^{\beta} \frac{1}{2} (r_2^2 - r_1^2) d\theta$$

 **Ex. 1** (example1, p636)


Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.

sol:

 **Ex. 2** (example2, p636)

Find the area inside the smaller loop of the limaçon $r = 2 \cos \theta + 1$.

sol:

 **Ex. 3** (example3, p637)

Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$.

sol:

實習課練習 (EXERCISE 11.5)

3. Find the area of the region inside the oval limaçon $r = 4 + 2 \cos \theta$.
8. Find the area of the region inside the six-leaved rose $r^2 = 2 \sin 3\theta$.
13. Find the area of the region inside the lemniscate $r^2 = 6 \cos 2\theta$ and outside the circle $r = \sqrt{3}$.
14. Find the area of the region inside the circle $r = 3a \cos \theta$ and outside the cardioid $r = a(1 + \cos \theta)$, $a > 0$.