THOMAS' CALCULUS (12/E)

14.3 Partial Derivatives

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Partial Derivatives of a Function of Two Variables

1.1 *Definition*

The _____ of $f(x,y)$ with respect to x at the point (x_0,y_0) is

$$= \_\_\_\_\_ \quad \_\_\_\_\_$$

provided the limit exists.

1.2 An equivalent expression

_____

1.3 Several notations

_____, _____, _____, _____, _____

1.4 The partial derivative of $f(x,y)$ with respect to _____ is obtained by differentiating f in the usual way while treating _____ as a _____.

✎ Ex. 1 ..... (example1, p766)

Find the values of $\partial f/\partial x$ and $\partial f/\partial y$ at the point $(4,-5)$ if $f(x,y)=x^2+3xy+y-1$.

sol:

Ex. 2 ..... (example2, p767)

Find $\partial f/\partial y$ as a function if $f(x,y) = y\sin xy$.

sol:

Ex. 3 ..... (example3, p767)

Find f_x and f_y as functions if $f(x,y) = \dfrac{2y}{y+\cos x}$.

sol:

Ex. 4 ..... (example4, p767)

Find $\partial z/\partial x$ if the equation $yz - \ln z = x + y$ defines z as a function of the two independent variables x and y and the partial derivative exists.

sol:

✎ Ex. **5** ..... (example6, p768)

Find $\partial f/\partial z$ if x, y and z are independent variables and $f(x, y, z) = x\sin(y + 3z)$.

sol:

2 Second-Order Partial Derivatives

2.1 Differentiate $f(x, y)$ twice, we produce its second-order derivatives.

_____, _____, _____, _____

2.2 *Theorem 2: The Mixed Derivative Theorem*

> If $f(x, y)$ and its partial derivatives _____ are defined throughout an open region containing a point (a, b) and are all continuous at (a, b), then _____.

✎ Ex. **6** ..... (example9, p770)

If $f(x, y) = x\cos y + ye^x$, find the second-order derivatives.

sol:

Ex. 7 ..... (example10, p770)

Find $\partial^2 w/\partial x \partial y$ if $w = xy + \dfrac{e^y}{y^2+1}$.

sol:

Ex. 8 ..... (example11, p771)

Find f_{yxyz} if $f(x, y, z) = 1 - 2xy^2z + x^2y$.

sol:

實習課練習 (EXERCISE 14.3)

7. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y) = \sqrt{x^2+y^2}$.

16. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y) = e^{xy}\ln y$.

21. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y) = \displaystyle\int_x^y g(t)\,dt$, ($g$ is continuous for all t).

22. Find $\partial f/\partial x$ and $\partial f/\partial y$: $f(x,y) = \displaystyle\sum_{n=1}^{\infty}(xy)^n$.

25. Find f_x, f_y and f_z: $f(x,y,z) = x - \sqrt{y^2+z^2}$.

32. Find f_x, f_y and f_z: $f(x,y,z) = e^{-xyz}$.

43. Find all the second-order partial derivatives: $g(x,y) = x^y + \cos y + y\sin x$.

52. Verify that $w_{xy} = w_{yx}$: $w = e^x + x\ln y + y\ln x$.

66. Find the value of $\partial x/\partial z$ at the point $(1,-1,-3)$ if the equation $xz+y\ln x - x^2+4=0$ defines x as a function of the two independent variables y and z and the partial dervitive exists.

72. Let $f(x,y) = \begin{cases} \sqrt{x}, & x \geq 0 \\ x^2, & x < 0. \end{cases}$

Find f_x, f_y, F_{xy} and f_{yx} and state the domain for each partial derivative.