THOMAS' CALCULUS (12/E)

14.4 The Chain Rule

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1 The Chain Rule

1.1 *Theorem 5: Chain Rule for Functions of Two Independent Variables*

If ______________ is differentiable and if ___________, ___________ are differentiable functions of t, then the composite __________________ is a differentiable function of t and

$$\frac{dw}{dt} = \underline{\hspace{5cm}}$$

1.2 *Theorem 6: Chain Rule for Functions of Three Independent Variables*

If $w = f(x, y, z)$ is differentiable and x, y, and z are differentiable functions of t, then the composite $w = f(x(t), y(t), z(t))$ is a differentiable function of t and

$$\frac{dw}{dt} = \underline{\hspace{7cm}}$$

1.3 The Branch Diagram:

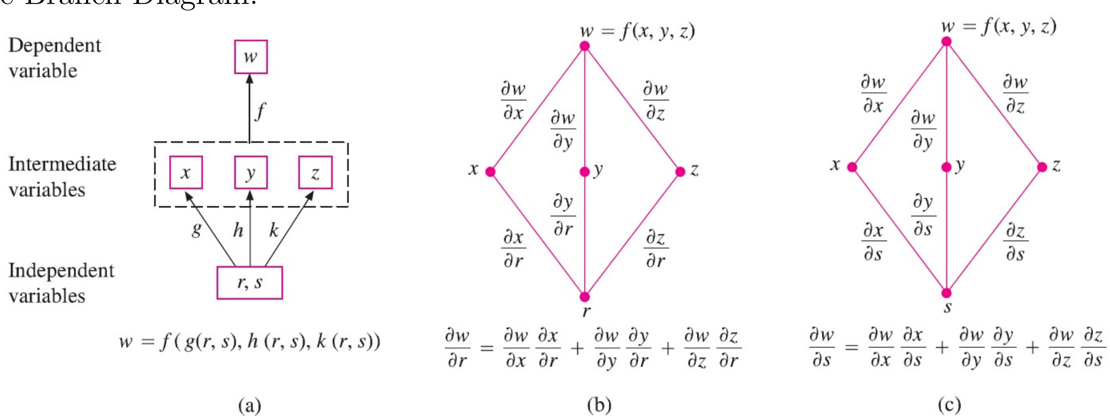

FIGURE 14.21 Composite function and branch diagrams for Theorem 7.

1.4 *Theorem 7: Chain Rule for Two Independent Variables and Three Intermediate Variables*

Suppose that $w = f(x, y, z)$, ______________, ______________ and ______________. If all four functions are differentiable, then w has partial derivatives with respect to r and s, given by

$$\frac{dw}{dr} = \underline{\hspace{6cm}}$$

$$\frac{dw}{ds} = \underline{\hspace{6cm}}$$

Ex. 1 .... (`example1, p776`)

Use the Chain Rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t$, $y = \sin t$. What is the derivative's value at $t = \pi/2$.

sol:

Ex. 2 .... (`example2, p777`)

Find dw/dt if $w = xy = z$, $x = \cos t$, $y = \sin t$, $z = t$.

sol:

Ex. 3 .... (`example3, p778`)

Express $\partial w/\partial r$ and $\partial w/\partial s$ interm of r and s if $w = x + 2y + z^2$, $x = \dfrac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$.

sol:

Ex. 4 .... (`example3, p779`)

Express $\partial w/\partial r$ and $\partial w/\partial s$ interm of r and s if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.

sol:

2 Implicit Differentiation

2.1 *Theorem 8: A Formula for Implicit Differentiation*

Suppose that $F(x, y)$ is differentiable and that the equation $F(x, y) = 0$ defines y as a differentiable function of x. Then at any point where $F_y \neq 0$,

$$\frac{dy}{dx} = \underline{\qquad\qquad}$$

pf.

2.2 Suppose that $F(x, y, z) = 0$ defines z implicitly as a function $z = f(x, y)$:

$$\frac{\partial z}{\partial x} = \underline{\qquad\qquad}, \quad \frac{\partial z}{\partial y} = \underline{\qquad\qquad}$$

✎ Ex. **5** .... (`example5, p780`)

Use implicit differentiation to find dy/dx if $y^2 - x^2 - \sin xy = 0$.

sol:

✎ Ex. **6** .... (`example6, p781`)

Find $\partial z/\partial x$ and $\partial z/\partial y$ at (0,0,0) if $x^3 + z^2 + ye^{xz} + z\cos y = 0$.

sol:

實習課練習 (EXERCISE 14.4)

3. Evaluate dw/dt at the given value of t: $w = \dfrac{x}{z} + \dfrac{y}{z}$, $x = \cos^2 t$, $y = \sin^2 t$, $z = 1/t$, $t = 3$.

5. Evaluate dw/dt at the given value of t: $w = 2ye^x - \ln z$, $x = \ln(t^2+1)$, $y = \tan^{-1} t$, $z = e^t$, $t = 1$.

8. Evaluate $\partial z/\partial u$ and $\partial z/\partial v$ at the given point (u, v): $z = \tan^{-1}(x/y)$, $x = u\cos v$, $y = u\sin v$, $(u, v) = (1.3, \pi/6)$.

9. Evaluate $\partial w/\partial u$ and $\partial w/\partial v$ at the given point (u, v): $w = xy + yz + xz$, $x = u + v$, $y = u - v$, $z = uv$, $(u, v) = (1/2, 1)$.

11. Evaluate $\partial u/\partial x$, $\partial u/\partial y$ and $\partial u/\partial z$ at the given point (x, y, z): $u = \dfrac{p-q}{q-r}$, $p = xy + yz + xz$, $q = x - y + z$, $r = x + y - z$, $(x, y, z) = (\sqrt{3}, 2, 1)$.

21. Draw a branch diagram and write a Chain Rule formula for each derivative. $\partial w/\partial s$ and $\partial w/\partial t$ for $w = g(u)$, $u = h(s, t)$.

28. Find dy/dx at the given point: $xe^y + \sin xy + y - \ln 2 = 0$, $(0, \ln 2)$

29. Find the values of $\partial z/\partial x$ and $\partial z/\partial y$ at the given point: $z^3 - xy + yz + y^3 - 2 = 0$, $(1, 1, 1)$

35. Find $\partial w/\partial v$ when $u = 0$, $v = 0$ if $w = x^2 + (y/x)$, $x = u - 2v + 1$, $y = 2u + v - 2$.