THOMAS' CALCULUS (12/E)

14.5 Directional Derivatives and Gradient Vectors

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Directional Derivatives in the Plane

1.1 Suppose that the function ____________ is defined throughout a region R in the xy-plane, that ______________ is a point in R, and that __________________ is a unit vector. Then the equations

$x =$ ____________, $y =$ ____________

parametrize the line through P_0 parallel to $\vec{u}$.

1.2 If the parameter s measures ______________ from P_0 in the direction of ______, we find the rate of change of f at P_0 in the direction of $\vec{u}$ by calculating __________ at _______.

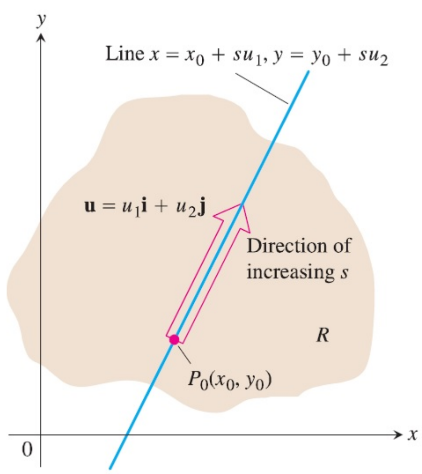

FIGURE 14.26 The rate of change of f in the direction of $\mathbf{u}$ at a point P_0 is the rate at which f changes along this line at P_0.

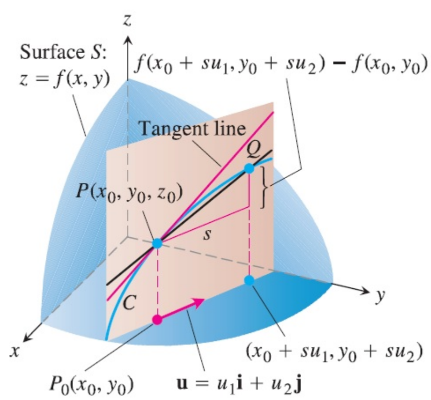

FIGURE 14.27 The slope of curve C at P_0 is $\lim_{Q \to P}$ slope (PQ); this is the directional derivative

$$\left(\frac{df}{ds}\right)_{\mathbf{u}, P_0} = (D_{\mathbf{u}} f)_{P_0}.$$

1.3 *Definition*

The derivative of f at $P_0(x_0, y_0)$ in the direction of the unit vector $\vec{u} = u_1\vec{i} + u_2\vec{j}$ is the number

$$\underline{\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

povided the limit exists.

1.4 The derivative of f at P_0 in the direction of $\vec{u}$ is also defined by ___________.

1.5 The partial defivatives $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ are the ______________________ of f at P_0 in the _____ and _____ directions.

Ex. 1 .... (`example1, p785`)

Using the definition, find the derivative of $f(x, y) = x^2 + xy$ at $P_0(1, 2)$ in the direction of the unit vector $\vec{u} = (1/\sqrt{2})\vec{i} + (1/\sqrt{2})\vec{j}$

sol:

2 Calculation and Gradients

2.1 *Definition*

The gradient vector (___________) of $f(x, y)$ at a point $P_0(x_0, y_0)$ is the vector

$$\underline{\qquad\qquad} = \underline{\qquad\qquad\qquad\qquad}$$

2.2 *Theorem 9: The Directional Derivative Is a Dot Product*

If $f(x, y)$ is differentiable in an open region containing $P_0(x_0, y_0)$, then

____

the dot product of _________________________ and ______.

Ex. **2** .... (example2, p786)

Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $\vec{v} = 3\vec{i} - 4\vec{j}$.

sol:

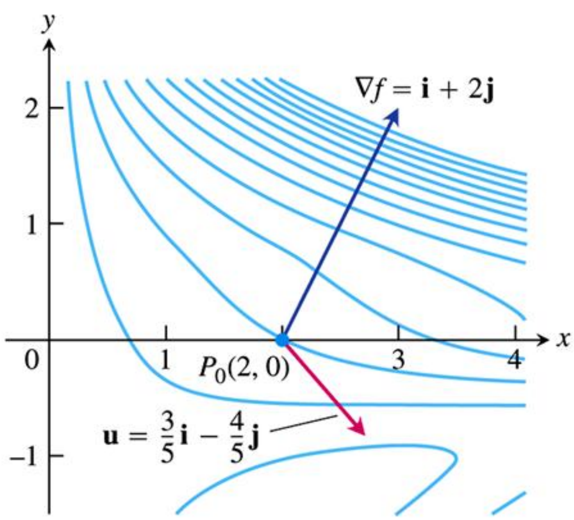

FIGURE 14.28 Picture ∇f as a vector in the domain of f. The figure shows a number of level curves of f. The rate at which f changes at (2, 0) in the direction $\mathbf{u} = (3/5)\mathbf{i} - (4/5)\mathbf{j}$ is $\nabla f \cdot \mathbf{u} = -1$ (Example 2).

實習課練習 (EXERCISE 14.5)

5. Find the gradient of the function at the given point. $f(x,y) = \sqrt{2x+3y}$, $(-1,2)$

9. Find ∇f at the given point: $f(x,y,z) = e^{x+y}\cos z + (y+1)\sin^{-1} x$, $(0,0,\pi/6)$.

☐ In Exercise 11-18, find the derivative of the function at P_0 in the direction of $\vec{u}$.

11. $f(x,y) = 2xy - 3y^2$, $P_0(5,5)$, $\vec{u} = 4\vec{i} + 3\vec{j}$.

13. $g(x,y) = \dfrac{x-y}{xy+2}$, $P_0(1,-1)$, $\vec{u} = 12\vec{i} + 5\vec{j}$.

15. $f(x,y,z) = xy + yz + zx$, $P_0(1,-1,2)$, $\vec{u} = 3\vec{i} + 6\vec{j} - 2\vec{k}$.

17. $g(x,y,z) = 3e^x \cos yz$, $P_0(0,0,0)$, $\vec{u} = 2\vec{i} + \vec{j} - 2\vec{k}$.