THOMAS' CALCULUS (12/E)

15.1 Double and Iterated Integrals over Rectangles

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Double Integrals

1.1 A rectangle with sides parallel to the coordinate axes: _____.

1.2 *Definition: The Double Integrals*

Let f be a function of two variables that is defined on a _____. If

_____

exists, we say that f is _____ on R. Moreover, _____, called the double integral of f over R, is then given by

$$\iint_R f(x,y)\,dA = \text{_____}$$

1.3 If $f(x) \geq 0$, _____ represents the _____ under the curve $y = f(x)$ between a and b.

1.4 If $f(x,y) \geq 0$, _____ represents the _____ under the surface $z = f(x,y)$ and above the rectangle R.

1.5 *Theorem: Fubini's Theorem (First Form)*

If $f(x,y)$ is continuous throughout the rectangular region $R : a \leq\leq b, c \leq y \leq d$, then

$$V = \underline{\qquad} = \underline{\qquad} = \underline{\qquad}$$

$$\underline{\qquad} = \underline{\qquad}.$$

The expression oh the right, called an iterated or repeated integral.

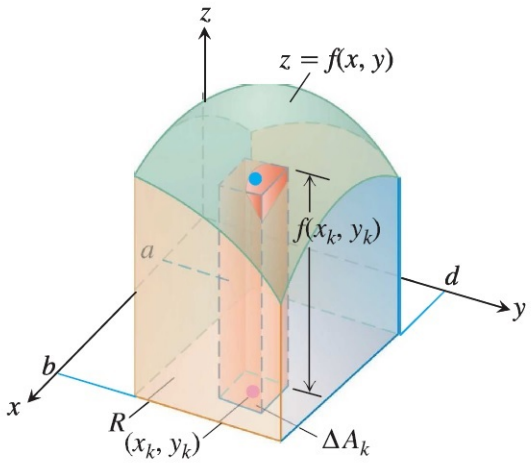
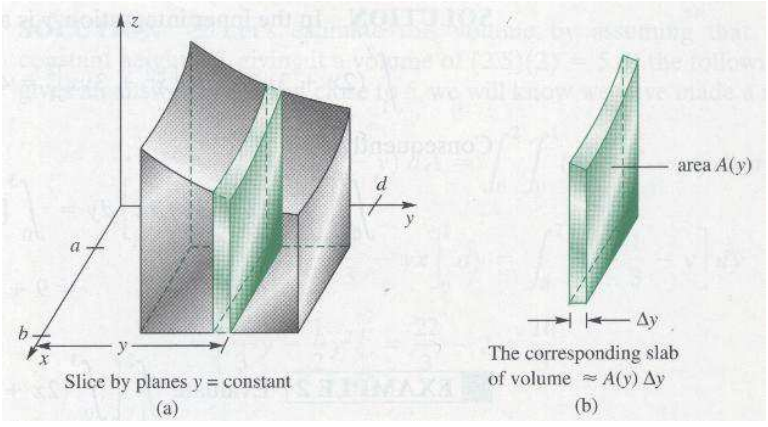
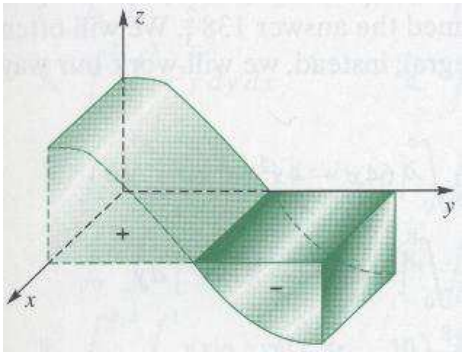


FIGURE 15.2 Approximating solids with rectangular boxes leads us to define the volumes of more general solids as double integrals. The volume of the solid shown here is the double integral of $f(x, y)$ over the base region R.

1.6 If $f(x,y)$ is $\underline{\qquad}$ on part of R, then $\iint_R f(x,y)\,dA$ gives the $\underline{\qquad}$ of the solid between the surface $z = f(x,y)$ and the rectangle R of the $xy-$plane. The actual volume of this slid is $\underline{\qquad}$.





Ex. 1 ..... (example1, p839)

Calculate $\iint_R f(x,y)\,dA$ for $f(x,y) = 100 - 6x^2y$ and $R : 0 \le x \le 2,\ -1 \le y \le 1$.

sol:

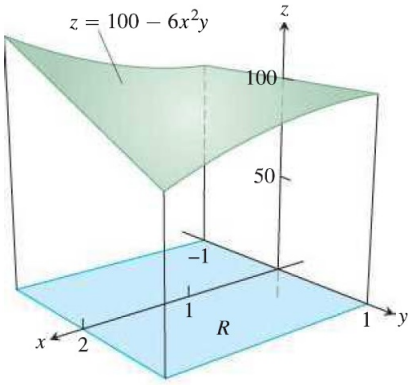


FIGURE 15.6 The double integral $\iint_R f(x,y)\,dA$ gives the volume under this surface over the rectangular region R (Example 1).

Ex. 2 ..... (example2, p840)

Find the volume of the region bounded above by the elliptical paraboloid $z = 10 + x^2 + 3y^2$ and below by the rectangle $R : 0 \le x \le 1, 0 \le y \le 2$.

sol:

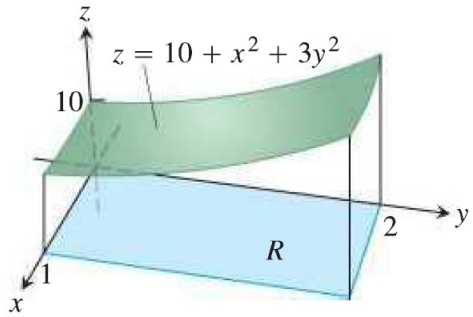


FIGURE 15.7 The double integral $\iint_R f(x,y)\,dA$ gives the volume under this surface over the rectangular region R (Example 2).

實習課練習 (EXERCISE 15.1)

4. $\int_0^1 \int_0^1 (1 - \frac{x^2+y^2}{2})\ dx\ dy$

7. $\int_0^1 \int_0^1 \frac{y}{1+xy}\ dx\ dy$

11. $\int_{-1}^2 \int_0^{\pi/2} y \sin x\ dx\ dy$

14. $\iint_R (\frac{\sqrt{x}}{y^2}) dA, \quad R : 0 \le x \le 4, 1 \le y \le 2.$

17. $\iint_R e^{x-y}, \quad R : 0 \le x \le \ln 2, 0 \le y \le 2.$

25. Find the volume of the region bounded above by the plane $z = 2 - x - y$ and below by the square $R : 0 \le x \le 1, 0 \le y \le 1$.