THOMAS' CALCULUS (12/E)

15.3 Area by Double Integration

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Areas of Bounded Regions in the Plane

1.1 If we take $f(x, y) = 1$ in the definition of the double integral over a region R, the Riemann sums reduce to

$$S_n = \underline{\qquad\qquad} = \underline{\qquad\qquad}.$$

This is the sum of areas of all small rectangles in the partition of R.

1.2 *Definition*

> The area of a closed, bounded plane region R is
> $$A = \underline{\qquad\qquad}$$

1.3 The average value of f over R:

✎ Ex. 1 ..... (example1, p850)

Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.

sol:

Ex. 2 ..... (example2, p851)

Find the area of the region R bounded by $y = x^2$ and the line $y = x + 2$.

sol:

Ex. 3 ..... (example3, p852)

Find the average value of $f(x, y) = x \cos xy$ over the rectangle $R : 0 \leq x \leq \pi, 0 \leq y \leq 1$.

sol:

實習課練習 (EXERCISE 15.3)

☐ In Exercise 1-12, express the region's area as an iterated double integral and evaluate the integral.

1. The coordinate axes and the line $x + y = 2$.

4. The parabola $x = y - y^2$ and the line $y = -x$.

14. $\int_0^3 \int_{-x}^{x(2-x)} dydx$.

18. $\int_0^2 \int_{x^2-4}^0 dydx + \int_0^4 \int_0^{\sqrt{x}} dydx$.

21. Find the average height of the paraboloid $z = x^2 + y^2$ over the square $0 \le x \le 2, 0 \le y \le 2$.

22. Find the average value of $f(x, y) = 1/(xy)$ over the square $\ln 2 \le x \le 2\ln 2, \ln 2 \le y \le 2\ln 2$