THOMAS' CALCULUS (12/E)

15.4 Double Integrals in Polar Form

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 Integrals in Polar Coordinates

1.1 Integrals are sometimes easier to evaluate if we change to ________.

1.2 Suppose that a function ________ is defined over a region R that is bounded by the rays ________ and ________, and by the continuous curves ________ and ________, where $0 \le g_1(\theta) \le g_2(\theta) \le a$, and $\alpha \le \theta \le \beta$.

圖示如下:

1.3 (a) The polar rectangles that lie inside R, calling their areas ________. Let ________ be any point in the polar rectangle whose area is ________.

(b) If f is continuous throughout R, the sum ________ will approach a limit as we refine the grid to make ________. The limit is called the double integral of f over R:

$$\lim_{n\to\infty} S_n = \underline{\qquad\qquad}$$

1.4 Write the sum S_n that expresses ΔA_k in terms of Δr and $\Delta\theta$.

圖示如下:

(a) Let the kth polar rectangle be ________. Let ______ be the average of the radii of the ________ and ____________ bounding the kth polar rectangle ΔA_k.

(b) The area of a wedge-shaped sector of a circle having radius r and angle θ: ______________.

(c) The area of the circular sectors:
Inner radius: __________________, Outer radius: __________________

(d) A_k =area of large sector − area of small sector

= ______________________________ = ________________________

(e) $S_n =$ ________________________. $\lim_{n\to\infty} S_n =$ ___________________.

1.5 The double integral of f over R is

$$\iint_R f(r,\theta)\ dA = \underline{\hspace{6cm}}$$

1.6 The area of a closed and bounded region R in the polar coordinate plane is

$$A = \underline{\hspace{5cm}}$$

1.7 *Changing Cartesian Integrals into Polar Integrals*

$$\iint_R f(x,y)\ dx\ dy = \underline{\hspace{7cm}},$$

where G denotes the same region of integration.

✎ **Ex. 1** .... (`example2, p855`)

Find the area enclosed by the lemniscate $r^2 = 4\cos 2\theta$.

sol:

✎ Ex. **2** .... (example3, p856)

Evaluate $\iint_R e^{x^2+y^2}\ dy\ dx$ where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

sol:

✎ Ex. **3** .... (example4, p856)

Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)\ dy\ dx$.

sol:

✎ Ex. **4** .... (example5, p856)

Find the volume of the solid region bounded above bby the paraboloid $z = 9 - x^2 - y^2$ and below by the unit circle in the xy-plane.

sol:

實習課練習 (EXERCISE 15.5)

9. $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dy\ dx.$

17. $\displaystyle\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\ dy\ dx$

19. $\displaystyle\int_{0}^{\ln 2}\int_{0}^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\ dx\ dy$

20. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{-\sqrt{1-y^2}} \ln(x^2+y^2+1)\ dx\ dy$

27. Find the area of the region cut from the first quadrant by the curve $r = 2(2 - \sin 2\theta)^{1/2}$

28. Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.