1.

(a)There is a point c on function f. If $\lim_{x \to c} f(x) = f(c)$,then we say that f is continuous at c.

(b)state: if f is Differentiable on c then f is continuous at c.

Proff: $\lim_{x \to c} f(x) = \lim_{x \to c} f(x) - f(c) + f(c)$

$$= \lim_{x \to c} f(x) - f(c) \times \frac{1}{x - c} \times (x - c) + f(c)$$

$$= \lim_{x \to c}(x - c) f'(c) + f(c) = f(c)$$

$\lim_{x \to c} f(x) = f(c)$ by definition, f is continuous on c

2.NO.

$$\lim_{x \to 1^+} \frac{x(x^2 - 1)}{|x^2 - 1|} = x = 1 \ but \ \lim_{x \to 1^-} \frac{x(x^2 - 1)}{|x^2 - 1|} = -x = -1$$

And

$$\lim_{x \to -1^+} \frac{x(x^2 - 1)}{|x^2 - 1|} = -x = 1 \ but \ \lim_{x \to -1^-} \frac{x(x^2 - 1)}{|x^2 - 1|} = x = -1$$

So $\lim_{x \to -1} f(x)$, $\lim_{x \to 1} f(x)$ *is not exist*.

We know that if $\lim_{x \to c} f(x) = f(c)$ then f is continuous on c.

But where $x = \pm 1$, limit does not exist.

So f can’t be extended to be continuous.

3.

(a)

$$\lim_{x\to 1}\frac{x^{50}-1}{x-1}=f'(1)=50\ where\ f(x)=x^{50}.$$

(b) $\lim_{x\to\infty}\frac{x+sinx+2\sqrt{x}}{x+sinx}=\frac{1+\frac{sinx}{x}+\frac{2}{\sqrt{x}}}{1+\frac{sinx}{x}}=1\ where\ \lim_{x\to\infty}\frac{sinx}{x}=0\ by\ Squeeze\ theorem.$

4.

$$\lim_{x\to -2^+}\frac{[2x]+2}{2x+2}=\frac{-4+2}{-2}=1\ but\ \lim_{x\to -2^-}\frac{[2x]+2}{2x+2}=\frac{-5+2}{-2}=\frac{3}{2}$$

So, $\lim_{x\to -2}\frac{[2x]+2}{2x+2}$ does not exist.

5.

let $y=f(x)+g(x)\ ,where\ f(x)=|x-1|, g(x)=\sin x.$

$f(x)\&g(x)\ is\ continuous\ for\ all\ x. so\ y=f(x)+g(x)\ is\ continuous\ for\ all\ x$

6.

$$f'(u)=\frac{df}{du}\times\frac{du}{dx}=2\left(\frac{2u-1}{2u+1}\right)\times\left(\frac{2(2u+1)-2(2u-1)}{(2u+1)^2}\right)\times\frac{du}{dx}$$

$$f'(g(x))=2\left(\frac{2(\frac{1}{x^2}-1)-1}{2(\frac{1}{x^2}-1)+1}\right)\times\left(\frac{4}{\left(2(\frac{1}{x^2}-1)+1\right)^2}\right)\left(\frac{-2}{x^3}\right), f'(g(-1))$$

$$=2\left(\frac{-1}{1}\right)\left(\frac{4}{2}\right)\left(\frac{-2}{-1}\right)=-16$$

7.

(a)

$$x\cos(2x+3y)=ysin5x$$

$$\rightarrow \cos(2x+3y) - xsin(2x+3y)\left(2+\frac{dy}{dx}\right) = \frac{\mathrm{dy}}{dx}(sin5x) + 5ycos5x$$

$$\cos(2x+3y) - 2xsin(2x+3y) - 5ycos5x = \frac{dy}{dx}\big(sin5x + xsin(2x+3y)\big)$$

$$\frac{\mathrm{dy}}{dx} = \frac{(\cos(2x+3y) - 2xsin(2x+3y) - 5ycos5x)}{sin5x + xsin(2x+3y)}$$

(b)

$$\mathrm{y}^2 = 1 - \frac{2}{x} \rightarrow 2y\frac{dy}{dx} = \frac{2}{x^2} \rightarrow \frac{dy}{dx} = \frac{1}{x^2y} \rightarrow \frac{d^2y}{dx^2} = \frac{2xy + x^2\frac{dy}{dx}}{(x^2y)^2}$$

8.

(a)

$$\mathrm{L}(x) = f(0) + f'(0)(x-0) = 1 + 2kx, where\ f'(x) = 2k(1+2x)^{k-1}$$

(b)

$(1+0.0002)^{50} = \mathrm{L}(1.0002) = f(0) + f'(0)(0.0002) = 1 + 50 \times 0.0002 =$

1.01

9.

$$\text{let } \mathrm{k}(x) = \frac{f(x)}{g(x)}, by\ definition\ k'(x) = \lim_{h\to 0}\frac{k(x+h) - k(x)}{h}$$

$$\mathrm{k}'(x) = \lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x+h)}{hg(x)g(x+h)}$$

$$= \lim_{h\to 0}\frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{hg(x)g(x+h)}$$

$$= \lim_{h\to 0}\frac{g(x)\big(f(x+h) - f(x)\big) - f(x)(g(x+h) - g(x))}{hg(x)g(x+h)}$$

$$= \frac{g(x)f'(x) - g'(x)f(x)}{g^2(x)}$$