LaTeX 作業

在「hmwu-LaTeX-Example.tex」檔案中，新增一附錄D「 LaTeX 作業 」，檔案更名為「座號-姓名-LaTeX-HW1.tex」，完成後上傳.tex和.pdf檔。

(1)

When the condition in the definition is satisfied, we say the Riemann sums of f on $[a, b]$ **converge** to the definite integral $J = \int_a^b f(x)\, dx$ and that f is **integrable** over $[a, b]$.

$$\lim_{\|P\|\to 0} \sum_{k=1} f(c_k)\, \Delta x_k = J = \int_a f(x)\, dx.$$

(2)

Half-Angle Formulas

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2} \tag{6}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2} \tag{7}$$

(3)

Solution

(a) Using the half-angle formula $\cos h = 1 - 2\sin^2(h/2)$, we calculate

$$\begin{aligned}
\lim_{h\to 0} \frac{\cos h - 1}{h} &= \lim_{h\to 0} - \frac{2\sin^2(h/2)}{h} \\
&= -\lim_{\theta\to 0} \frac{\sin\theta}{\theta} \sin\theta \\
&= -(1)(0) = 0.
\end{aligned}$$

(4)

3.5 Instead of a calendar of 365 days, we have one with just 12 months. Let C_n be the event n arbitrary persons have different months of birth. Then

$$\mathrm{P}(C_3) = \left(1 - \frac{2}{12}\right) \cdot \left(1 - \frac{1}{12}\right) = \frac{55}{72} = 0.7639$$

and it is no surprise that this is much smaller than $\mathrm{P}(B_3)$. The general formula is

$$\mathrm{P}(C_n) = \left(1 - \frac{n-1}{12}\right) \cdots \left(1 - \frac{2}{12}\right) \cdot \left(1 - \frac{1}{12}\right).$$

Note that it is correct even if n is 13 or more, in which case $\mathrm{P}(C_n) = 0$.

(5)

Definition. A discrete random variable X has a *Poisson distribution* with parameter μ, where $\mu > 0$ if its probability mass function p is given by

$$p(k) = \mathrm{P}(X = k) = \frac{\mu^k}{k!}\,\mathrm{e}^{-\mu} \quad \text{for } k = 0, 1, 2, \ldots.$$

We denote this distribution by $Pois(\mu)$.

(6)

Definition. A continuous random variable has a *normal distribution* with parameters μ and $\sigma^2 > 0$ if its probability density function f is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for } -\infty < x < \infty.$$

We denote this distribution by $N(\mu, \sigma^2)$.

(7)

$$\det \begin{vmatrix} c_0 & c_1 & \cdots & c_n \\ c_1 & c_2 & \cdots & c_{n+1} \\ \vdots & \vdots & & \vdots \\ c_n & c_{n+1} & \cdots & c_{2n} \end{vmatrix} \leq 0$$

(8)

Prove that $\lim_{x \to 2} f(x) = 4$ if $f(x) = \begin{cases} x^2, & x \neq 2. \\ 1, & x = 2. \end{cases}$