國立臺北大學 109 學年度第 2 學期 期末 考試命題紙

考試科目： 微積分　　開課班別： 通訊 1　　命題教授: 吳漢銘

考試日期：06 月 21 日 (一) 10:10~11:40	※准帶項目打「O」，否則打「×」					1. 需加發計算紙或答案紙請在試題內封袋備註。 2. 為環保節能減碳，試題一律採雙面印刷，如有特殊印製需求，請註記：
本試題共 1 頁，印刷份數: 60 份	計算機	課本	筆記	電子辭典	紙本字典	
備註：注意事項要看!! (§10.4~§15.4)	×	×	×	×	×	

注意事項: (1) 答案卷請寫上姓名及學號。(2) 請按題號順序書寫。(3) 每一題號需置於答案卷最左邊。(4) 可用鉛筆。(5) 需要計算過程。(6) 請好好拍照上傳。(7) 每題 10 分，總分共 100 分。(8) 請記得寫「宣誓詞」。

1. Determine the convergence of the series: $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot\cdots\cdot(2n-1)}{[2\cdot 4\cdot\cdots\cdot(2n)](3^n+1)}$.

2. Find the Taylor series generated by $\dfrac{1}{1-2x}$ at $x=1$.

3. Determine the convergence of the series. (converge absolutely, converge conditionally, or diverge)
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n^2+1)}{2n^2+n-1}$. (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(x-1)^{2n+1}}{2n+1}$

4. Graph the polar equation $r = 1+2\sin\theta$.

5. Find the area inside the cardioid $r = 2(1+\sin\theta)$ and outside the circle $r = 2\sin\theta$.

6. Let $f(x,y) = \begin{cases} y^3, & y \geq 0 \\ -y^2, & y<0 \end{cases}$ Find f_x, f_y, f_{xy} and f_{yx} and state the domain for each partial derivative.

7. Let
$$f(x,y) = \begin{cases} \dfrac{\sin(x-y)}{|x|+|y|}, & |x|+|y| \neq 0, \\ 0, & (x,y)=(0,0). \end{cases}$$
   Is f continuous at the origin? Why?

8. Assuming that the equation $2xy + e^{x+y} - 2 = 0$ define y as a differentiable function of x, using implicit differentiation to find the value of dy/dx at point $P(0, \ln 2)$.

9. Evaluate integrals: (a) $\displaystyle\int_0^2 \int_0^{4-x^2} 2x\, dydx$. (b) $\displaystyle\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{2\pi\sin\pi x^2}{x^2}\, dxdy$.

10. Using double integrals to find the area of the "triangular" region in the xy-plane that is bounded on the right by the parabola $y = x^2$, on the left by the line $x + y = 2$, and above by the line $y = 2$.